PROBLEM 2.1

A nylon thread is subjected to a 8.5-N tension force. Knowing that $E = 3.3\text{ GPa}$ and that the length of the thread increases by 1.1%, determine (*a*) the diameter of the thread, (*b*) the stress in the thread.

SOLUTION

(*a*) Strain: $\varepsilon = \dfrac{\delta}{L} = \dfrac{1.1}{100} = 0.011$

Stress: $\sigma = E\varepsilon = (3.3 \times 10^9)(0.011) = 36.3 \times 10^6\text{ Pa}$

$$\sigma = \frac{P}{A}$$

Area: $A = \dfrac{P}{\sigma} = \dfrac{8.5}{36.3 \times 10^6} = 234.16 \times 10^{-9}\text{ m}^2$

Diameter: $d = \sqrt{\dfrac{4A}{\pi}} = \sqrt{\dfrac{(4)(234.16 \times 10^{-9})}{\pi}} = 546 \times 10^{-6}\text{ m}$ $\quad d = 0.546\text{ mm}$ ◀

(*b*) Stress: $\quad \sigma = 36.3\text{ MPa}$ ◀

PROBLEM 2.2

A 4.8-ft-long steel wire of $\frac{1}{4}$ -in.-diameter is subjected to a 750-lb tensile load. Knowing that $E = 29 \times 10^6$ psi, determine (*a*) the elongation of the wire, (*b*) the corresponding normal stress.

SOLUTION

(*a*) Deformation: $\delta = \dfrac{PL}{AE}; \qquad A = \dfrac{\pi d^2}{4}$

Area: $A = \dfrac{\pi(0.25 \text{ in.})^2}{4} = 4.9087 \times 10^{-2} \text{ in}^2$

$$\delta = \frac{(750 \text{ lb})(4.8 \text{ ft} \times 12 \text{ in./ft})}{(4.9087 \times 10^{-2} \text{ in}^2)(29 \times 10^6 \text{ psi})}$$

$\delta = 3.0347 \times 10^{-2} \text{ in.}$ $\qquad \delta = 0.0303 \text{ in.}$ ◀

(*b*) Stress: $\sigma = \dfrac{P}{A}$

Area: $\sigma = \dfrac{(750 \text{ lb})}{(4.9087 \times 10^{-2} \text{ in}^2)}$

$\sigma = 1.52790 \times 10^4 \text{ psi}$ $\qquad \sigma = 15.28 \text{ ksi}$ ◀

PROBLEM 2.3

An 18-m-long steel wire of 5-mm diameter is to be used in the manufacture of a prestressed concrete beam. It is observed that the wire stretches 45 mm when a tensile force **P** is applied. Knowing that $E = 200$ GPa, determine (*a*) the magnitude of the force **P**, (*b*) the corresponding normal stress in the wire.

SOLUTION

(*a*) $\delta = \dfrac{PL}{AE}$, or $P = \dfrac{\delta AE}{L}$

with $A = \dfrac{1}{4}\pi d^2 = \dfrac{1}{4}\pi(0.005)^2 = 19.6350 \times 10^{-6}\ \text{m}^2$

$$P = \frac{(0.045\ \text{m})(19.6350\times 10^{-6}\ \text{m}^2)(200\times 10^9\ \text{N/m}^2)}{18\ \text{m}} = 9817.5\ \text{N}$$

$P = 9.82$ kN ◀

(*b*) $\sigma = \dfrac{P}{A} = \dfrac{9817.5\ \text{N}}{19.6350\times 10^{-6}\ \text{m}^2} = 500\times 10^6\ \text{Pa}$ $\sigma = 500$ MPa ◀

PROBLEM 2.4

Two gage marks are placed exactly 250 mm apart on a 12-mm-diameter aluminum rod with $E = 73$ GPa and an ultimate strength of 140 MPa. Knowing that the distance between the gage marks is 250.28 mm after a load is applied, determine (*a*) the stress in the rod, (*b*) the factor of safety.

SOLUTION

(*a*) $\delta = L - L_0$

$= 250.28 \text{ mm} - 250 \text{ mm}$

$= 0.28 \text{ mm}$

$$\varepsilon = \frac{\delta}{L_0}$$

$$= \frac{0.28 \text{ mm}}{250 \text{ mm}}$$

$= 1.11643 \times 10^{-4}$

$\sigma = E\varepsilon$

$= (73 \times 10^9 \text{ Pa})(1.11643 \times 10^{-4})$

$= 8.1760 \times 10^7 \text{ Pa}$

$\sigma = 81.8 \text{ MPa}$ ◀

(*b*) $$\text{F.S.} = \frac{\sigma_u}{\sigma}$$

$$= \frac{140 \text{ MPa}}{81.760 \text{ MPa}}$$

$= 1.71233$ $\text{F.S.} = 1.712$ ◀

PROBLEM 2.5

An aluminum pipe must not stretch more than 0.05 in. when it is subjected to a tensile load. Knowing that $E = 10.1 \times 10^6$ psi and that the maximum allowable normal stress is 14 ksi, determine (*a*) the maximum allowable length of the pipe, (*b*) the required area of the pipe if the tensile load is 127.5 kips.

SOLUTION

(*a*) $\delta = \dfrac{PL}{AE}$

Thus, $L = \dfrac{EA\delta}{P} = \dfrac{E\delta}{\sigma} = \dfrac{(10.1 \times 10^6)(0.05)}{14 \times 10^3}$

$L = 36.1 \text{ in.}$ ◀

(*b*) $\sigma = \dfrac{P}{A}$

Thus, $A = \dfrac{P}{\sigma} = \dfrac{127.5 \times 10^3}{14 \times 10^3}$ $A = 9.11 \text{ in}^2$ ◀

PROBLEM 2.6

A control rod made of yellow brass must not stretch more than 3 mm when the tension in the wire is 4 kN. Knowing that E = 105 GPa and that the maximum allowable normal stress is 180 MPa, determine (*a*) the smallest diameter rod that should be used, (*b*) the corresponding maximum length of the rod.

SOLUTION

(*a*) $\sigma = \frac{P}{A}; \quad A = \frac{\pi d^2}{4}$

Substituting, we have

$$\sigma = \frac{P}{\left(\frac{\pi d^2}{4}\right)} \quad \Rightarrow \quad d = \sqrt{\frac{4P}{\sigma\pi}}$$

$$d = \sqrt{\frac{4(4 \times 10^3 \text{ N})}{(180 \times 10^6 \text{ Pa})\pi}}$$

$$d = 5.3192 \times 10^{-3} \text{ m}$$

$d = 5.32$ mm ◀

(*b*) $\sigma = E\varepsilon; \quad \varepsilon = \frac{\delta}{L}$

Substituting, we have

$$\sigma = E\frac{\delta}{L} \quad \Rightarrow \quad L = \frac{E\partial}{\sigma}$$

$$L = \frac{(105 \times 10^9 \text{ Pa})(3 \times 10^{-3} \text{ m})}{(180 \times 10^6 \text{ Pa})}$$

$L = 1.750$ m ◀

PROBLEM 2.7

A steel control rod is 5.5 ft long and must not stretch more than 0.04 in. when a 2-kip tensile load is applied to it. Knowing that $E = 29 \times 10^6$ psi, determine (*a*) the smallest diameter rod that should be used, (*b*) the corresponding normal stress caused by the load.

SOLUTION

(*a*) $\delta = \frac{PL}{AE}: \quad 0.04 \text{ in.} = \frac{(2000 \text{ lb})(5.5 \times 12 \text{ in.})}{A(29 \times 10^6 \text{ psi})}$

$$A = \frac{1}{4}\pi d^2 = 0.113793 \text{ in}^2$$

$$d = 0.38063 \text{ in.}$$

$d = 0.381$ in. ◀

(*b*) $\sigma = \frac{P}{A} = \frac{2000 \text{ lb}}{0.113793 \text{ in}^2} = 17575.8 \text{ psi}$

$\sigma = 17.58$ ksi ◀

PROBLEM 2.8

A cast-iron tube is used to support a compressive load. Knowing that $E = 10 \times 10^6$ psi and that the maximum allowable change in length is 0.025%, determine (*a*) the maximum normal stress in the tube, (*b*) the minimum wall thickness for a load of 1600 lb if the outside diameter of the tube is 2.0 in.

SOLUTION

(*a*) $$\frac{\delta}{L} = \frac{\delta}{100} = 0.00025$$

$$\sigma = E\varepsilon; \qquad \varepsilon = \frac{\delta}{L}$$

$$\therefore \qquad \sigma = E\frac{\delta}{L}$$

$$\sigma = (10 \times 10^6\,\text{psi})(0.00025)$$

$$\sigma = 2.50 \times 10^3 \text{ psi}$$

$$\sigma = 2.50 \text{ ksi} \blacktriangleleft$$

(*b*) $$\sigma = \frac{P}{A}; \qquad \therefore \qquad A = \frac{P}{\sigma} = \frac{1600 \text{ lb}}{2.50 \times 10^3 \text{ psi}} = 0.64 \text{ in}^2$$

$$A = \frac{\pi}{4}\left(d_o^2 - d_i^2\right)$$

$$d_i^2 = d_o^2 - \frac{4A}{\pi}$$

$$d_i^2 = (2.0 \text{ in.})^2 - \frac{4(0.64 \text{ in}^2)}{\pi} = 3.1851 \text{ in}^2$$

$$\therefore \qquad d_i = 1.78469 \text{ in.}$$

$$t = \frac{1}{2}(d_o - d_i) = \frac{1}{2}(2.0 \text{ in.} - 1.78469 \text{ in.})$$

$$t = 0.107655 \text{ in.}$$

$$t = 0.1077 \blacktriangleleft$$

PROBLEM 2.9

A 4-m-long steel rod must not stretch more than 3 mm and the normal stress must not exceed 150 MPa when the rod is subjected to a 10-kN axial load. Knowing that $E = 200$ GPa, determine the required diameter of the rod.

SOLUTION

$L = 4$ m

$\delta = 3 \times 10^{-3}$ m, $\quad \sigma = 150 \times 10^{6}$ Pa

$E = 200 \times 10^{9}$ Pa, $\quad P = 10 \times 10^{3}$ N

Stress: $\sigma = \dfrac{P}{A}$

$$A = \frac{P}{\sigma} = \frac{10 \times 10^{3}\ \text{N}}{150 \times 10^{6}\ \text{Pa}} = 66.667 \times 10^{-6}\text{m}^2 = 66.667\ \text{mm}^2$$

Deformation: $\delta = \dfrac{PL}{AE}$

$$A = \frac{PL}{E\delta} = \frac{(10 \times 10^{3})(4)}{(200 \times 10^{9})(3 \times 10^{-3})} = 66.667 \times 10^{-6}\text{m}^2 = 66.667\ \text{mm}^2$$

The larger value of A governs: $A = 66.667\ \text{mm}^2$

$$A = \frac{\pi}{4}d^2 \quad d = \sqrt{\frac{4A}{\pi}} = \sqrt{\frac{4(66.667)}{\pi}}$$

$d = 9.21$ mm ◀

PROBLEM 2.10

A nylon thread is to be subjected to a 10-N tension. Knowing that $E = 3.2$ GPa, that the maximum allowable normal stress is 40 MPa, and that the length of the thread must not increase by more than 1%, determine the required diameter of the thread.

SOLUTION

Stress criterion:

$$\sigma = 40 \text{ MPa} = 40 \times 10^6 \text{ Pa} \quad P = 10 \text{ N}$$

$$\sigma = \frac{P}{A}: \quad A = \frac{P}{\sigma} = \frac{10 \text{ N}}{40 \times 10^6 \text{ Pa}} = 250 \times 10^{-9} \text{m}^2$$

$$A = \frac{\pi}{4} d^2: \quad d = 2\sqrt{\frac{A}{\pi}} = 2\sqrt{\frac{250 \times 10^{-9}}{\pi}} = 564.19 \times 10^{-6} \text{m}$$

$$d = 0.564 \text{ mm}$$

Elongation criterion:

$$\frac{\delta}{L} = 1\% = 0.01$$

$$\delta = \frac{PL}{AE}:$$

$$A = \frac{P/E}{\delta/L} = \frac{10 \text{ N}/3.2 \times 10^9 \text{ Pa}}{0.01} = 312.5 \times 10^{-9} \text{ m}^2$$

$$d = 2\sqrt{\frac{A}{\pi}} = 2\sqrt{\frac{312.5 \times 10^{-9}}{\pi}} = 630.78 \times 10^{-6} \text{m}^2$$

$$d = 0.631 \text{ mm}$$

The required diameter is the larger value: $d = 0.631$ mm ◀

PROBLEM 2.11

A block of 10-in. length and 1.8 × 1.6-in. cross section is to support a centric compressive load **P**. The material to be used is a bronze for which $E = 14 \times 10^6$ psi. Determine the largest load that can be applied, knowing that the normal stress must not exceed 18 ksi and that the decrease in length of the block should be at most 0.12% of its original length.

SOLUTION

Considering allowable stress, $\sigma = 18 \text{ ksi} \quad \text{or} \quad 18 \times 10^3 \text{psi}$

Cross-sectional area: $A = (1.8 \text{ in.})(1.6 \text{ in.}) = 2.880 \text{ in}^2$

$$\sigma = \frac{P}{A} \quad \Rightarrow \quad P = \sigma A$$

$$= (18 \times 10^3 \text{psi})(2.880 \text{ in}^2)$$

$$= 5.1840 \times 10^4 \text{ lb}$$

or 51.840 kips

Considering allowable deformation, $\dfrac{\delta}{L} = 0.12\% \quad \text{or} \quad 0.0012 \text{ in.}$

$$\delta = \frac{PL}{AE} \quad \Rightarrow \quad P = AE\left(\frac{\delta}{L}\right)$$

$$P = (2.880 \text{ in}^2)(14 \times 10^6 \text{ psi})(0.0012 \text{ in.})$$

$$= 4.8384 \times 10^4 \text{ lb}$$

or 48.384 kips

The smaller value for **P** resulting from the required deformation criteria governs.

48.4 kips ◀

PROBLEM 2.12

A square yellow-brass bar must not stretch more than 2.5 mm when it is subjected to a tensile load. Knowing that $E = 105$ GPa and that the allowable tensile strength is 180 MPa, determine (*a*) the maximum allowable length of the bar, (*b*) the required dimensions of the cross section if the tensile load is 40 kN.

SOLUTION

$\sigma = 180 \times 10^6$ Pa $\quad P = 40 \times 10^3$ N

$E = 105 \times 10^9$ Pa $\quad \delta = 2.5 \times 10^{-3}$ m

(*a*) $\delta = \dfrac{PL}{AE} = \dfrac{\sigma L}{E}$

$$L = \frac{E\delta}{\sigma} = \frac{(105\times10^9)(2.5\times10^{-3})}{180\times10^6} = 1.45833 \text{ m}$$

$L = 1.458$ m ◀

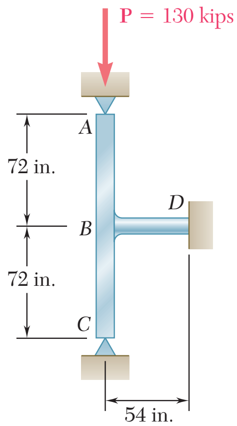
(*b*) $\sigma = \dfrac{P}{A}$

$$A = \frac{P}{\sigma} = \frac{40 \times 10^3}{180 \times 10^6} = 222.22 \times 10^{-6}\ \text{m}^2 = 222.22\ \text{mm}^2$$

$A = a^2 \quad a = \sqrt{A} = \sqrt{222.22}$

$a = 14.91$ mm ◀

PROBLEM 2.13



Rod BD is made of steel $(E = 29 \times 10^6 \text{ psi})$ and is used to brace the axially compressed member ABC. The maximum force that can be developed in member BD is $0.02P$. If the stress must not exceed 18 ksi and the maximum change in length of BD must not exceed 0.001 times the length of ABC, determine the smallest-diameter rod that can be used for member BD.

SOLUTION

$$F_{BD} = 0.02P = (0.02)(130) = 2.6 \text{ kips} = 2.6 \times 10^3 \text{ lb}$$

Considering stress, $\sigma = 18 \text{ ksi} = 18 \times 10^3 \text{ psi}$

$$\sigma = \frac{F_{BD}}{A} \qquad \therefore \quad A = \frac{F_{BD}}{\sigma} = \frac{2.6}{18} = 0.14444 \text{ in}^2$$

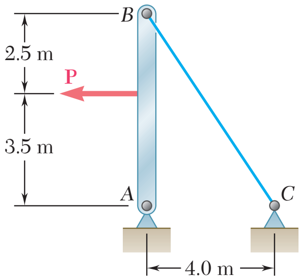
Considering deformation, $\delta = (0.001)(144) = 0.144 \text{ in.}$

$$\delta = \frac{F_{BD}L_{BD}}{AE} \qquad \therefore \quad A = \frac{F_{BD}L_{BD}}{E\delta} = \frac{(2.6 \times 10^3)(54)}{(29 \times 10^6)(0.144)} = 0.03362 \text{ in}^2$$

Larger area governs. $A = 0.14444 \text{ in}^2$

$$A = \frac{\pi}{4}d^2 \qquad \therefore \quad d = \sqrt{\frac{4A}{\pi}} = \sqrt{\frac{(4)(0.14444)}{\pi}}$$

$d = 0.429 \text{ in.}$ ◀



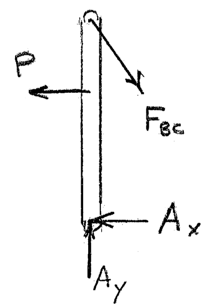
PROBLEM 2.14

The 4-mm-diameter cable BC is made of a steel with $E = 200$ GPa. Knowing that the maximum stress in the cable must not exceed 190 MPa and that the elongation of the cable must not exceed 6 mm, find the maximum load **P** that can be applied as shown.

SOLUTION

$$L_{BC} = \sqrt{6^2 + 4^2} = 7.2111 \text{ m}$$

Use bar AB as a free body.



$$+\circlearrowleft \Sigma M_A = 0: \quad 3.5P - (6)\left(\frac{4}{7.2111}F_{BC}\right) = 0$$

$$P = 0.9509F_{BC}$$

Considering allowable stress, $\sigma = 190 \times 10^6$ Pa

$$A = \frac{\pi}{4}d^2 = \frac{\pi}{4}(0.004)^2 = 12.566 \times 10^{-6} \text{ m}^2$$

$$\sigma = \frac{F_{BC}}{A} \quad \therefore \quad F_{BC} = \sigma A = (190 \times 10^6)(12.566 \times 10^{-6}) = 2.388 \times 10^3 \text{ N}$$

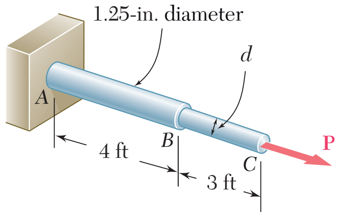
Considering allowable elongation, $\delta = 6 \times 10^{-3}$ m

$$\delta = \frac{F_{BC}L_{BC}}{AE} \quad \therefore \quad F_{BC} = \frac{AE\delta}{L_{BC}} = \frac{(12.566 \times 10^{-6})(200 \times 10^9)(6 \times 10^{-3})}{7.2111} = 2.091 \times 10^3 \text{ N}$$

Smaller value governs. $F_{BC} = 2.091 \times 10^3$ N

$$P = 0.9509F_{BC} = (0.9509)(2.091 \times 10^3) = 1.988 \times 10^3 \text{ N}$$

$P = 1.988$ kN ◀



PROBLEM 2.15

A single axial load of magnitude $P = 15$ kips is applied at end C of the steel rod ABC. Knowing that $E = 30 \times 10^6$ psi, determine the diameter d of portion BC for which the deflection of point C will be 0.05 in.

SOLUTION

$$\delta_C = \sum \frac{PL_i}{A_i E_i} = \left(\frac{PL}{AE}\right)_{AB} + \left(\frac{PL}{AE}\right)_{BC}$$

$$L_{AB} = 4 \text{ ft} = 48 \text{ in.}; \qquad L_{BC} = 3 \text{ ft} = 36 \text{ in.}$$

$$A_{AB} = \frac{\pi d^2}{4} = \frac{\pi(1.25 \text{ in.})^2}{4} = 1.22718 \text{ in}^2$$

Substituting, we have

$$0.05 \text{ in.} = \left(\frac{15 \times 10^3 \text{ lb}}{30 \times 10^6 \text{psi}}\right)\left(\frac{48 \text{ in.}}{1.22718 \text{ in}^2} + \frac{36 \text{ in.}}{A_{BC}}\right)$$

$$A_{BC} = 0.59127 \text{ in}^2$$

$$A_{BC} = \frac{\pi d^2}{4}$$

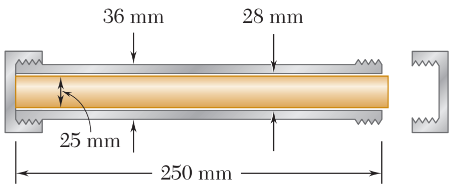
or $$d = \sqrt{\frac{4A_{BC}}{\pi}}$$

$$d = \sqrt{\frac{4(0.59127 \text{ in}^2)}{\pi}}$$

$$d = 0.86766 \text{ in.}$$

$d = 0.868$ in. ◀

PROBLEM 2.16



A 250-mm-long aluminum tube ($E = 70$ GPa) of 36-mm outer diameter and 28-mm inner diameter can be closed at both ends by means of single-threaded screw-on covers of 1.5-mm pitch. With one cover screwed on tight, a solid brass rod ($E = 105$ GPa) of 25-mm diameter is placed inside the tube and the second cover is screwed on. Since the rod is slightly longer than the tube, it is observed that the cover must be forced against the rod by rotating it one-quarter of a turn before it can be tightly closed. Determine (*a*) the average normal stress in the tube and in the rod, (*b*) the deformations of the tube and of the rod.

SOLUTION

$$A_{\text{tube}} = \frac{\pi}{4}(d_o^2 - d_i^2) = \frac{\pi}{4}(36^2 - 28^2) = 402.12 \text{ mm}^2 = 402.12 \times 10^{-6} \text{ m}^2$$

$$A_{\text{rod}} = \frac{\pi}{4}d^2 = \frac{\pi}{4}(25)^2 = 490.87 \text{ mm}^2 = 490.87 \times 10^{-6} \text{ m}^2$$

$$\delta_{\text{tube}} = \frac{PL}{E_{\text{tube}}A_{\text{tube}}} = \frac{P(0.250)}{(70 \times 10^9)(402.12 \times 10^{-6})} = 8.8815 \times 10^{-9} P$$

$$\delta_{\text{rod}} = -\frac{PL}{E_{\text{rod}}A_{\text{rod}}} = \frac{P(0.250)}{(105 \times 10^6)(490.87 \times 10^{-6})} = -4.8505 \times 10^{-9} P$$

$$\delta^* = \left(\frac{1}{4}\text{ turn}\right) \times 1.5 \text{ mm} = 0.375 \text{ mm} = 375 \times 10^{-6} \text{ m}$$

$$\delta_{\text{tube}} = \delta^* + \delta_{\text{rod}} \quad \text{or} \quad \delta_{\text{tube}} - \delta_{\text{rod}} = \delta^*$$

$$8.8815 \times 10^{-9} P + 4.8505 \times 10^{-9} P = 375 \times 10^{-6}$$

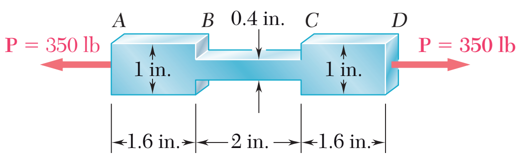
$$P = \frac{0.375 \times 10^{-3}}{(8.8815 + 4.8505)(10^{-9})} = 27.308 \times 10^3 \text{ N}$$

(*a*) $$\sigma_{\text{tube}} = \frac{P}{A_{\text{tube}}} = \frac{27.308 \times 10^3}{402.12 \times 10^{-6}} = 67.9 \times 10^6 \text{ Pa}$$ $\sigma_{\text{tube}} = 67.9$ MPa ◀

$$\sigma_{\text{rod}} = -\frac{P}{A_{\text{rod}}} = -\frac{27.308 \times 10^3}{490.87 \times 10^{-6}} = -55.6 \times 10^6 \text{ Pa}$$ $\sigma_{\text{rod}} = -55.6$ MPa ◀

(*b*) $$\delta_{\text{tube}} = (8.8815 \times 10^{-9})(27.308 \times 10^3) = 242.5 \times 10^{-6} \text{ m}$$ $\delta_{\text{tube}} = 0.243$ mm ◀

$$\delta_{\text{rod}} = -(4.8505 \times 10^{-9})(27.308 \times 10^3) = -132.5 \times 10^{-6} \text{ m}$$ $\delta_{\text{rod}} = -0.1325$ mm ◀



PROBLEM 2.17

The specimen shown has been cut from a $\frac{1}{4}$-in.-thick sheet of vinyl ($E = 0.45 \times 10^6$ psi) and is subjected to a 350-lb tensile load. Determine (*a*) the total deformation of the specimen, (*b*) the deformation of its central portion *BC*.

SOLUTION

$$\delta_{AB} = \frac{PL_{AB}}{EA_{AB}} = \frac{(350\text{ lb})(1.6\text{ in.})}{(0.45\times10^6\text{ psi})(1\text{ in.})(0.25\text{ in.})} = 4.9778\times10^{-3}\text{ in.}$$

$$\delta_{BC} = \frac{PL_{BC}}{EA_{BC}} = \frac{(350\text{ lb})(2\text{ in.})}{(0.45\times10^6\text{ psi})(0.4\text{ in.})(0.25\text{ in.})} = 15.5556\times10^{-3}\text{ in.}$$

$$\delta_{CD} = \delta_{AB} = 4.9778\times10^{-3}\text{ in.}$$

(*a*) Total deformation:

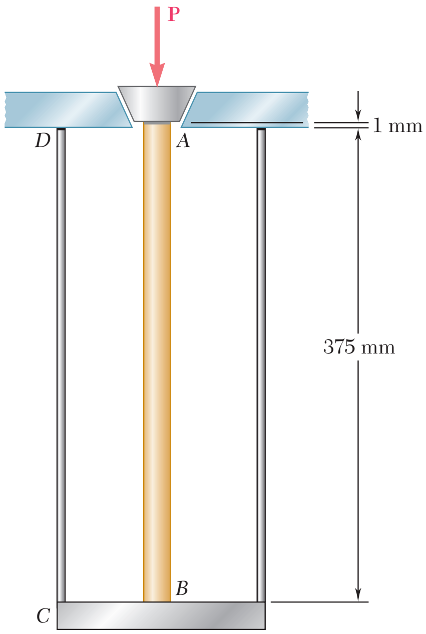
$$\delta = \delta_{AB} + \delta_{BC} + \delta_{CD}$$

$$\delta = 25.511\times10^{-3}\text{ in.}$$

$$\delta = 25.5\times10^{-3}\text{ in.} \blacktriangleleft$$

(*b*) Deformation of portion *BC* :

$$\delta_{BC} = 15.56\times10^{-3}\text{ in.} \blacktriangleleft$$



PROBLEM 2.18

The brass tube AB ($E = 105$ GPa) has a cross-sectional area of 140 mm^2 and is fitted with a plug at A. The tube is attached at B to a rigid plate that is itself attached at C to the bottom of an aluminum cylinder ($E = 72$ GPa) with a cross-sectional area of 250 mm^2. The cylinder is then hung from a support at D. In order to close the cylinder, the plug must move down through 1 mm. Determine the force **P** that must be applied to the cylinder.

SOLUTION

Shortening of brass tube AB:

$$L_{AB} = 375 + 1 = 376 \text{ mm} = 0.376 \text{ m} \quad A_{AB} = 140 \text{ mm}^2 = 140 \times 10^{-6} \text{ m}^2$$

$$E_{AB} = 105 \times 10^9 \text{ Pa}$$

$$\delta_{AB} = \frac{PL_{AB}}{E_{AB}A_{AB}} = \frac{P(0.376)}{(105 \times 10^9)(140 \times 10^{-6})} = 25.578 \times 10^{-9} P$$

Lengthening of aluminum cylinder CD:

$$L_{CD} = 0.375 \text{ m} \quad A_{CD} = 250 \text{ mm}^2 = 250 \times 10^{-6} \text{ m}^2 \quad E_{CD} = 72 \times 10^9 \text{ Pa}$$

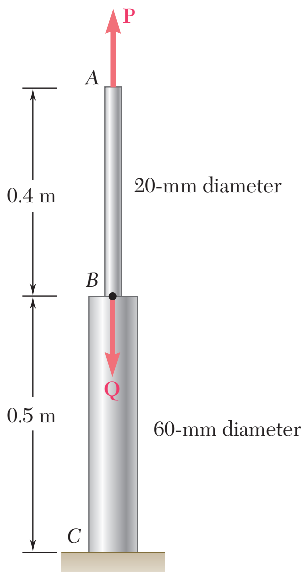
$$\delta_{CD} = \frac{PL_{CD}}{E_{CD}A_{CD}} = \frac{P(0.375)}{(72 \times 10^9)(250 \times 10^{-6})} = 20.833 \times 10^{-9} P$$

Total deflection: $\delta_A = \delta_{AB} + \delta_{CD}$ where $\delta_A = 0.001$ m

$$0.001 = (25.578 \times 10^{-9} + 20.833 \times 10^{-9})P$$

$$P = 21.547 \times 10^3 \text{ N}$$

$P = 21.5$ kN ◀



PROBLEM 2.19

Both portions of the rod *ABC* are made of an aluminum for which $E = 70$ GPa. Knowing that the magnitude of **P** is 4 kN, determine (*a*) the value of **Q** so that the deflection at *A* is zero, (*b*) the corresponding deflection of *B*.

SOLUTION

(*a*) $A_{AB} = \frac{\pi}{4} d_{AB}^2 = \frac{\pi}{4}(0.020)^2 = 314.16 \times 10^{-6}\,\text{m}^2$

$A_{BC} = \frac{\pi}{4} d_{BC}^2 = \frac{\pi}{4}(0.060)^2 = 2.8274 \times 10^{-3}\,\text{m}^2$

Force in member *AB* is *P* tension.

Elongation:

$$\delta_{AB} = \frac{PL_{AB}}{EA_{AB}} = \frac{(4 \times 10^3)(0.4)}{(70 \times 10^9)(314.16 \times 10^{-6})} = 72.756 \times 10^{-6}\,\text{m}$$

Force in member *BC* is $Q - P$ compression.

Shortening:

$$\delta_{BC} = \frac{(Q-P)L_{BC}}{EA_{BC}} = \frac{(Q-P)(0.5)}{(70 \times 10^9)(2.8274 \times 10^{-3})} = 2.5263 \times 10^{-9}(Q-P)$$

For zero deflection at *A*, $\delta_{BC} = \delta_{AB}$

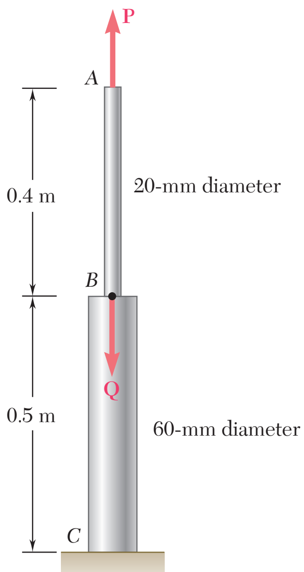
$$2.5263 \times 10^{-9}(Q-P) = 72.756 \times 10^{-6} \quad \therefore \quad Q - P = 28.8 \times 10^3\,\text{N}$$

$$Q = 28.3 \times 10^3 + 4 \times 10^3 = 32.8 \times 10^3\,\text{N}$$

$Q = 32.8$ kN ◀

(*b*) $\delta_{AB} = \delta_{BC} = \delta_B = 72.756 \times 10^{-6}\,\text{m}$

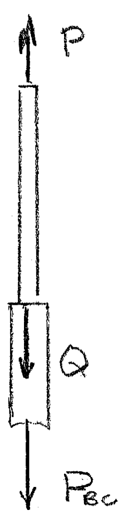
$\delta_{AB} = 0.0728\ \text{mm} \downarrow$ ◀



PROBLEM 2.20

The rod *ABC* is made of an aluminum for which $E = 70$ GPa. Knowing that $\mathbf{P} = 6$ kN and $\mathbf{Q} = 42$ kN, determine the deflection of (*a*) point *A*, (*b*) point *B*.

SOLUTION



$$A_{AB} = \frac{\pi}{4} d_{AB}^2 = \frac{\pi}{4}(0.020)^2 = 314.16 \times 10^{-6}\,\text{m}^2$$

$$A_{BC} = \frac{\pi}{4} d_{BC}^2 = \frac{\pi}{4}(0.060)^2 = 2.8274 \times 10^{-3}\,\text{m}^2$$

$$P_{AB} = P = 6 \times 10^3\,\text{N}$$

$$P_{BC} = P - Q = 6 \times 10^3 - 42 \times 10^3 = -36 \times 10^3\,\text{N}$$

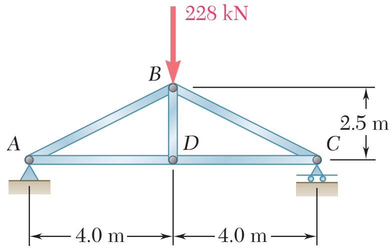
$$L_{AB} = 0.4\text{ m} \quad L_{BC} = 0.5\text{ m}$$

$$\delta_{AB} = \frac{P_{AB}L_{AB}}{A_{AB}E_A} = \frac{(6 \times 10^3)(0.4)}{(314.16 \times 10^{-6})(70 \times 10^9)} = 109.135 \times 10^{-6}\,\text{m}$$

$$\delta_{BC} = \frac{P_{BC}L_{BC}}{A_{BC}E} = \frac{(-36 \times 10^3)(0.5)}{(2.8274 \times 10^{-3})(70 \times 10^9)} = -90.947 \times 10^{-6}\,\text{m}$$

(*a*) $\delta_A = \delta_{AB} + \delta_{BC} = 109.135 \times 10^{-6} - 90.947 \times 10^{-6}\,\text{m} = 18.19 \times 10^{-6}\,\text{m}$ $\qquad \delta_A = 0.01819\text{ mm}\uparrow$ ◀

(*b*) $\delta_B = \delta_{BC} = -90.9 \times 10^{-6}\,\text{m} = -0.0909\text{ mm}$ $\qquad$ or $\qquad \delta_B = 0.0909\text{ mm}\downarrow$ ◀



PROBLEM 2.21

For the steel truss ($E = 200$ GPa) and loading shown, determine the deformations of the members AB and AD, knowing that their cross-sectional areas are 2400 mm^2 and 1800 mm^2, respectively.

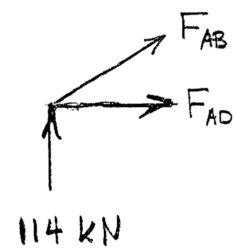
SOLUTION

Statics: Reactions are 114 kN upward at A and C.

Member BD is a zero force member.

$$L_{AB} = \sqrt{4.0^2 + 2.5^2} = 4.717 \text{ m}$$

Use joint A as a free body.



$$+\uparrow \Sigma F_y = 0: \ 114 + \frac{2.5}{4.717} F_{AB} = 0$$

$$F_{AB} = -215.10 \text{ kN}$$

$$\xrightarrow{+} \Sigma F_x = 0: \ F_{AD} + \frac{4}{4.717} F_{AB} = 0$$

$$F_{AD} = -\frac{(4)(-215.10)}{4.717} = 182.4 \text{ kN}$$

Member AB:

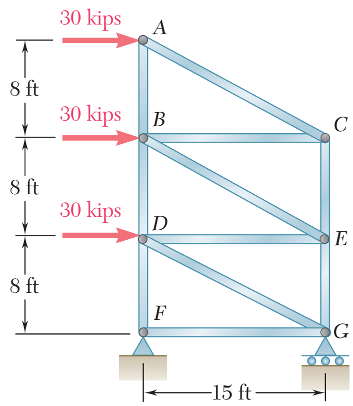
$$\delta_{AB} = \frac{F_{AB} L_{AB}}{E A_{AB}} = \frac{(-215.10 \times 10^3)(4.717)}{(200 \times 10^9)(2400 \times 10^{-6})}$$

$$= -2.11 \times 10^{-3} \text{ m} \qquad \delta_{AB} = -2.11 \text{ mm} \blacktriangleleft$$

Member AD:

$$\delta_{AD} = \frac{F_{AD} L_{AD}}{E A_{AD}} = \frac{(182.4 \times 10^3)(4.0)}{(200 \times 10^9)(1800 \times 10^{-6})}$$

$$= 2.03 \times 10^{-3} \text{ m} \qquad \delta_{AD} = 2.03 \text{ mm} \blacktriangleleft$$

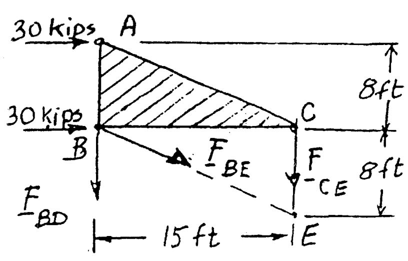


PROBLEM 2.22

For the steel truss $(E = 29 \times 10^6 \text{ psi})$ and loading shown, determine the deformations of the members *BD* and *DE*, knowing that their cross-sectional areas are 2 in^2 and 3 in^2, respectively.

SOLUTION

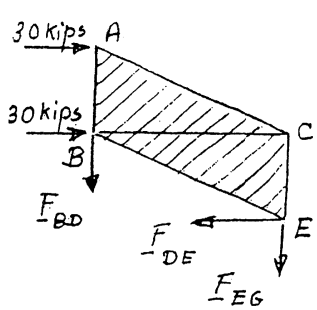
Free body: Portion *ABC* of truss



$$+\circlearrowleft \Sigma M_E = 0: \; F_{BD}(15 \text{ ft}) - (30 \text{ kips})(8 \text{ ft}) - (30 \text{ kips})(16 \text{ ft}) = 0$$

$$F_{BD} = +48.0 \text{ kips}$$

Free body: Portion *ABEC* of truss

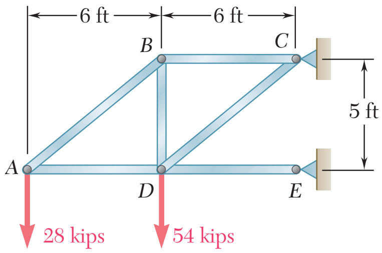


$$\xrightarrow{+} \Sigma F_x = 0: \; 30 \text{ kips} + 30 \text{ kips} - F_{DE} = 0$$

$$F_{DE} = +60.0 \text{ kips}$$

$$\delta_{BD} = \frac{PL}{AE} = \frac{(+48.0 \times 10^3 \text{ lb})(8 \times 12 \text{ in.})}{(2 \text{ in}^2)(29 \times 10^6 \text{ psi})} \qquad \delta_{BD} = +79.4 \times 10^{-3} \text{ in.} \blacktriangleleft$$

$$\delta_{DE} = \frac{PL}{AE} = \frac{(+60.0 \times 10^3 \text{ lb})(15 \times 12 \text{ in.})}{(3 \text{ in}^2)(29 \times 10^6 \text{ psi})} \qquad \delta_{DE} = +124.1 \times 10^{-3} \text{ in.} \blacktriangleleft$$



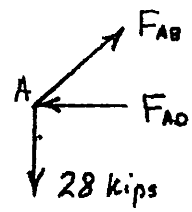
PROBLEM 2.23

Members *AB* and *BC* are made of steel $(E = 29 \times 10^6 \text{psi})$ with cross-sectional areas of 0.80 in^2 and 0.64 in^2, respectively. For the loading shown, determine the elongation of (*a*) member *AB*, (*b*) member *BC*.

SOLUTION

(*a*) $L_{AB} = \sqrt{6^2 + 5^2} = 7.810 \text{ ft} = 93.72 \text{ in.}$

Use joint *A* as a free body.



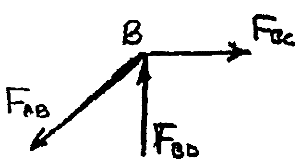
$$+\uparrow \Sigma F_y = 0: \quad \frac{5}{7.810} F_{AB} - 28 = 0$$

$$F_{AB} = 43.74 \text{ kip} = 43.74 \times 10^3 \text{ lb}$$

$$\delta_{AB} = \frac{F_{AB} L_{AB}}{E A_{AB}} = \frac{(43.74 \times 10^3)(93.72)}{(29 \times 10^6)(0.80)}$$

$\delta_{AB} = 0.1767$ in. ◀

(*b*) Use joint *B* as a free body.



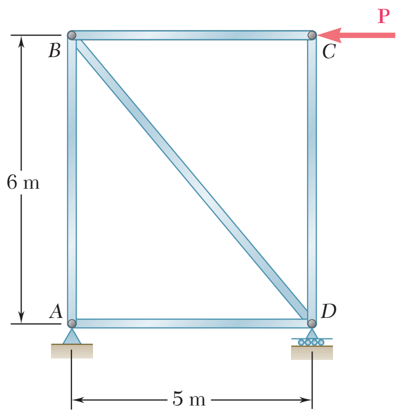
$$\xrightarrow{+} \Sigma F_x = 0: \quad F_{BC} - \frac{6}{7.810} F_{AB} = 0$$

$$F_{BC} = \frac{(6)(43.74)}{7.810} = 33.60 \text{ kip} = 33.60 \times 10^3 \text{ lb}$$

$$\delta_{BC} = \frac{F_{BC} L_{BC}}{E A_{BC}} = \frac{(33.60 \times 10^3)(72)}{(29 \times 10^6)(0.64)}$$

$\delta_{BC} = 0.1304$ in. ◀

PROBLEM 2.24



The steel frame (E = 200 GPa) shown has a diagonal brace BD with an area of 1920 mm^2. Determine the largest allowable load **P** if the change in length of member BD is not to exceed 1.6 mm.

SOLUTION

$$\delta_{BD} = 1.6 \times 10^{-3}\text{ m}, \quad A_{BD} = 1920\text{ mm}^2 = 1920 \times 10^{-6}\text{ m}^2$$

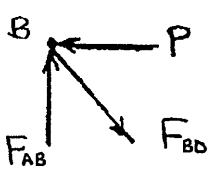
$$L_{BD} = \sqrt{5^2 + 6^2} = 7.810\text{ m}, \quad E_{BD} = 200 \times 10^9\text{ Pa}$$

$$\delta_{BD} = \frac{F_{BD}L_{BD}}{E_{BD}A_{BD}}$$

$$F_{BD} = \frac{E_{BD}A_{BD}\delta_{BD}}{L_{BD}} = \frac{(200 \times 10^9)(1920 \times 10^{-6})(1.6 \times 10^{-3})}{7.81}$$

$$= 78.67 \times 10^3\text{ N}$$

Use joint B as a free body. $\xrightarrow{+} \Sigma F_x = 0$:

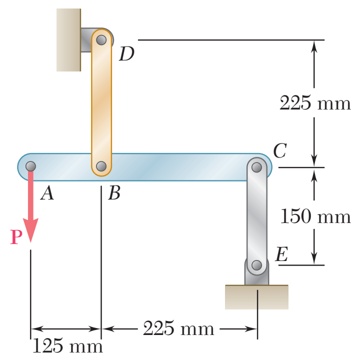


$$\frac{5}{7.810}F_{BD} - P = 0$$

$$P = \frac{5}{7.810}F_{BD} = \frac{(5)(78.67 \times 10^3)}{7.810}$$

$$= 50.4 \times 10^3\text{ N}$$

$P = 50.4$ kN ◀

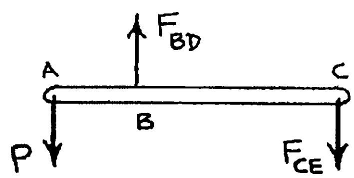


PROBLEM 2.25

Link BD is made of brass $(E = 105 \text{ GPa})$ and has a cross-sectional area of 240 mm^2. Link CE is made of aluminum $(E = 72 \text{ GPa})$ and has a cross-sectional area of 300 mm^2. Knowing that they support rigid member ABC, determine the maximum force **P** that can be applied vertically at point A if the deflection of A is not to exceed 0.35 mm.

SOLUTION

Free body member AC:

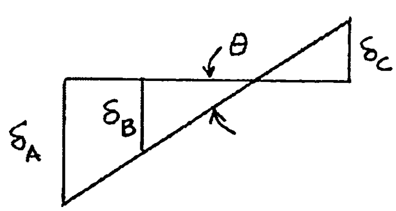


$$+\circlearrowleft \Sigma M_C = 0: \; 0.350P - 0.225F_{BD} = 0$$
$$F_{BD} = 1.55556P$$
$$+\circlearrowleft \Sigma M_B = 0: \; 0.125P - 0.225F_{CE} = 0$$
$$F_{CE} = 0.55556P$$

$$\delta_B = \delta_{BD} = \frac{F_{BD}\,L_{BD}}{E_{BD}A_{BD}} = \frac{(1.55556P)(0.225)}{(105\times 10^9)(240\times 10^{-6})} = 13.8889\times 10^{-9}P$$
$$\delta_C = \delta_{CE} = \frac{F_{CE}L_{CE}}{E_{CE}A_{CE}} = \frac{(0.55556P)(0.150)}{(72\times 10^9)(300\times 10^{-6})} = 3.8581\times 10^{-9}P$$

Deformation Diagram:

From the deformation diagram,

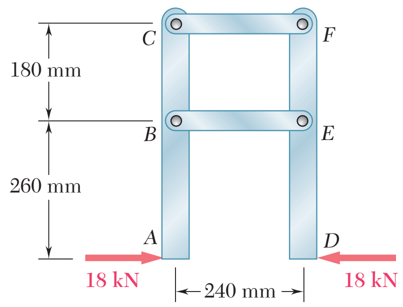


Slope: $$\theta = \frac{\delta_B + \delta_C}{L_{BC}} = \frac{17.7470\times 10^{-9}P}{0.225} = 78.876\times 10^{-9}P$$
$$\delta_A = \delta_B + L_{AB}\theta$$
$$= 13.8889\times 10^{-9}P + (0.125)(78.876\times 10^{-9}P)$$
$$= 23.748\times 10^{-9}P$$

Apply displacement limit. $\delta_A = 0.35\times 10^{-3} \text{ m} = 23.748\times 10^{-9}P$

$$P = 14.7381\times 10^3 \text{ N}$$

$P = 14.74$ kN ◀

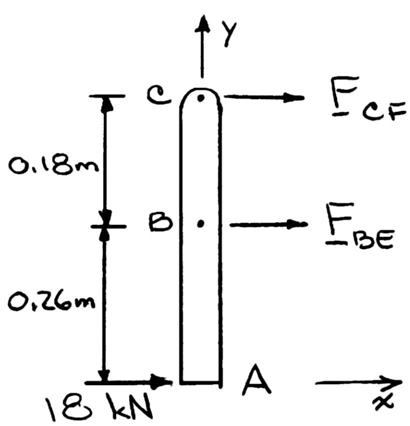


PROBLEM 2.26

Members *ABC* and *DEF* are joined with steel links (E = 200 GPa). Each of the links is made of a pair of 25 × 35-mm plates. Determine the change in length of (*a*) member *BE*, (*b*) member *CF*.

SOLUTION

Free body diagram of Member *ABC*:



$$+\circlearrowleft \Sigma M_B = 0:$$

$$(0.26 \text{ m})(18 \text{ kN}) - (0.18 \text{ m}) F_{CF} = 0$$

$$F_{CF} = 26.0 \text{ kN}$$

$$+\longrightarrow \Sigma F_x = 0:$$

$$18 \text{ kN} + F_{BE} + 26.0 \text{ kN} = 0$$

$$F_{BE} = -44.0 \text{ kN}$$

Area for link made of two plates:

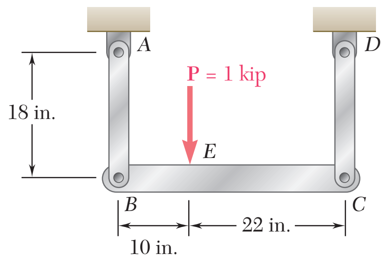
$$A = 2(0.025 \text{ m})(0.035 \text{ m}) = 1.750 \times 10^{-3} \text{ m}^2$$

(*a*) $$\delta_{BE} = \frac{F_{BE}L}{EA} = \frac{(-44.0 \times 10^3 \text{ N})(0.240 \text{ m})}{(200 \times 10^9 \text{ Pa})(1.75 \times 10^{-3} \text{ m}^2)} = -30.171 \times 10^{-6} \text{ m}$$

$$\delta_{BE} = -0.0302 \text{ mm} \blacktriangleleft$$

(*b*) $$\delta_{CF} = \frac{F_{BF}L}{EA} = \frac{(26.0 \times 10^3 \text{ N})(0.240 \text{ m})}{(200 \times 10^9 \text{ Pa})(1.75 \times 10^{-3} \text{ m}^2)} = 17.8286 \times 10^{-6} \text{ m}$$

$$\delta_{CF} = 0.01783 \text{ mm} \blacktriangleleft$$

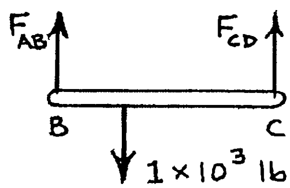


PROBLEM 2.27

Each of the links AB and CD is made of aluminum ($E = 10.9 \times 10^6$ psi) and has a cross-sectional area of 0.2 in^2. Knowing that they support the rigid member BC, determine the deflection of point E.

SOLUTION

Free body BC:



$$+\circlearrowleft \Sigma M_C = 0:\ -(32)F_{AB} + (22)(1\times 10^3) = 0$$

$$F_{AB} = 687.5 \text{ lb}$$

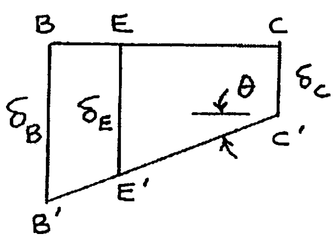
$$+\uparrow \Sigma F_y = 0:\ 687.5 - 1\times 10^3 + F_{CD} = 0$$

$$F_{CD} = 312.5 \text{ lb}$$

$$\delta_{AB} = \frac{F_{AB}\,L_{AB}}{EA} = \frac{(687.5)(18)}{(10.9\times 10^6)(0.2)} = 5.6766\times 10^{-3} \text{ in.} = \delta_B$$

$$\delta_{CD} = \frac{F_{CD}\,L_{CD}}{EA} = \frac{(312.5)(18)}{(10.9\times 10^6)(0.2)} = 2.5803\times 10^{-3} \text{ in.} = \delta_C$$

Deformation diagram:



$$\text{Slope } \theta = \frac{\delta_B - \delta_C}{L_{BC}} = \frac{3.0963\times 10^{-3}}{32}$$

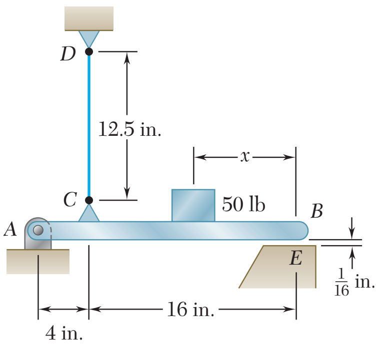
$$= 96.759\times 10^{-6} \text{ rad}$$

$$\delta_E = \delta_C + L_{EC}\theta$$

$$= 2.5803\times 10^{-3} + (22)(96.759\times 10^{-6})$$

$$= 4.7090\times 10^{-3} \text{ in.}$$

$$\delta_E = 4.71\times 10^{-3} \text{ in.} \downarrow \blacktriangleleft$$



PROBLEM 2.28

The length of the $\frac{3}{32}$-in.-diameter steel wire *CD* has been adjusted so that with no load applied, a gap of $\frac{1}{16}$ in. exists between the end *B* of the rigid beam *ACB* and a contact point *E*. Knowing that $E = 29 \times 10^6$ psi, determine where a 50-lb block should be placed on the beam in order to cause contact between *B* and *E*.

SOLUTION

Rigid beam *ACB* rotates through angle θ to close gap.

$$\theta = \frac{1/16}{20} = 3.125 \times 10^{-3} \text{ rad}$$

Point *C* moves downward.

$$\delta_C = 4\theta = 4(3.125 \times 10^{-3}) = 12.5 \times 10^{-3} \text{ in.}$$

$$\delta_{CD} = \delta_C = 12.5 \times 10^{-3} \text{ in.}$$

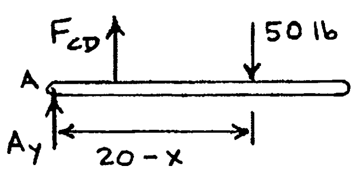
$$A_{CD} = \frac{\pi}{d} d^2 = \frac{\pi}{4}\left(\frac{3}{32}\right)^2 = 6.9029 \times 10^{-3} \text{ in}^2$$

$$\delta_{CD} = \frac{F_{CD} L_{CD}}{E A_{CD}}$$

$$F_{CD} = \frac{E A_{CD} \delta_{CD}}{L_{CD}} = \frac{(29 \times 10^6)(6.9029 \times 10^{-3})(12.5 \times 10^{-3})}{12.5}$$

$$= 200.18 \text{ lb}$$

Free body *ACB*:



$$+\circlearrowleft \Sigma M_A = 0: \quad 4F_{CD} - (50)(20 - x) = 0$$

$$20 - x = \frac{(4)(200.18)}{50} = 16.0144$$

$$x = 3.9856 \text{ in.}$$

For contact, $x < 3.99$ in. ◀